

# Design and Validation of Involute Spur Gear with Asymmetric Teeth to Improve Bending Load Carrying Capacity

<sup>1</sup>Mr. H. B. Borhade, <sup>2</sup>Dr. R. R. Navthar

<sup>1</sup>PG Scholar, Mechanical Engineering Dept. P.D.V.V.P.C.O.E. Ahmednagar, Maharashtra, India

<sup>2</sup>Professor Dept. of Mech Engg. P.D.V.V.P.C.O.E. Ahmednagar, Maharashtra, India

---

**Abstract:** Gear is a machine element used to transmit motion and power between rotating shafts by means of progressive engagement of projections called teeth. Gears are classified according to the relative position of the axes of the shaft, type of gearing, peripheral velocity of the gears and position of teeth on gear surface. Presently gears are suffered by backlash the amount by which the width of a tooth space exceeds the thickness of the engaging tooth on the pitch circles, undercut a condition in generated gear teeth when any part of the fillet curve lies inside of a line drawn tangent to the working profile at its lowest point and interference is an important aspect of kinematics of gearing. When the gear tooth tries to dig below the base circle of mating gear then the gear tooth action shall be non conjugate and violate the fundamental law of gearing this non conjugate action is called the interference. These defects can be eliminated by increasing the pressure angle, by increasing the addendum of mating gear and another way of increasing the load capacity of transmissions is to modify the involute geometry. This has been a standard practice in sophisticated gear design for many years; the nomenclature describing these types of gear modifications can be quite confusing with reference to addendum modification or profile shift. An additional alteration that is very rarely used is to make the gears asymmetric with different pressure angles for each side of the tooth. In this study, geometric design of asymmetric gear involute meshing is done. This method is appropriate for the geometric design of spur gears with a small teeth number and is based on the generalized model of involute meshing.

**Keywords:** Pressure Angle, Gear Tooth Profile, Involute Meshing.

---

## I. INTRODUCTION

### 1.1 Gear Technology:

Gears are the most common means of transmitting power in the modern mechanical engineering world. They vary from a tiny size used in watches to the large gears used in lifting mechanisms and speed reducers. They form vital elements of main and ancillary mechanisms in many machines such as automobiles, tractors, metal cutting machine tools etc. Toothed gears are used to change the speed and power ratio as well as direction between input and output.

### 1.2 Law of Gearing:

A primary requirement of gears is the constancy of angular velocities or proportionality of position transmission. Precision instruments require positioning fidelity. High-speed and/or high-power gear trains also require transmission at constant angular velocities in order to avoid severe dynamic problems. Constant velocity (i.e., constant ratio) motion transmission is defined as "conjugate action" of the gear tooth profiles. A geometric relationship can be derived for the form of the tooth profiles to provide conjugate action, which is summarized as the Law of Gearing as follows:

"A common normal to the tooth profiles at their point of contact must, in all positions of the contacting teeth, pass through a fixed point on the line-of-centers called the pitch point."

Any two curves or profiles engaging each other and satisfying the law of gearing are conjugate curves.

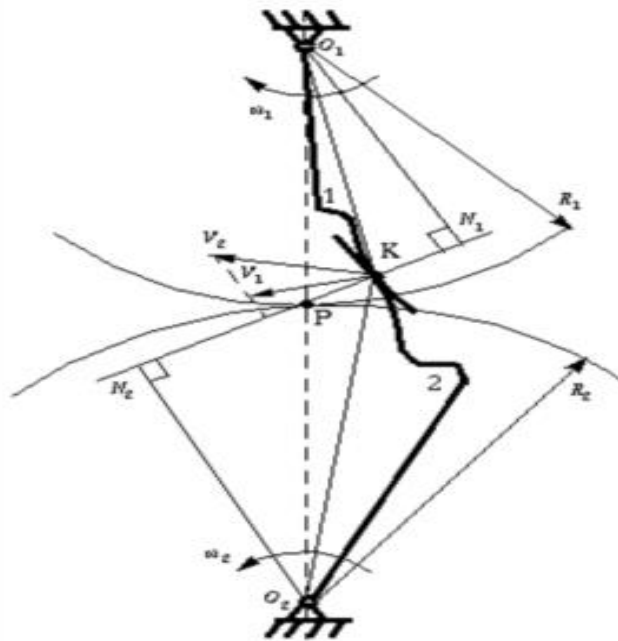


Fig. 1.1 Law of Gearing

Fig 1.1 shows two mating gear teeth, in which

- Tooth profile 1 drives tooth profile 2 by acting at the instantaneous contact point K.
- $N_1N_2$  is the common normal of the two profiles.
- $N_1$  is the foot of the perpendicular from  $O_1$  to  $N_1N_2$
- $N_2$  is the foot of the perpendicular from  $O_2$  to  $N_1N_2$ .

Although the two profiles have different velocities  $V_1$  and  $V_2$  at point K, their velocities along  $N_1N_2$  are equal in both magnitude and direction. Otherwise the two tooth profiles would separate from each other. Therefore, it is noticed that the intersection of the tangency  $N_1N_2$  and the line of center  $O_1O_2$  is at point P, and thus, the relationship between the angular velocities of the driving gear to the driven gear, or velocity ratio, of a pair of mating teeth is Point P is very important to the velocity ratio, and it is called the pitch point. Pitch point divides the line between the line of centers and its position decides the velocity ratio of the two teeth. The above expression is the fundamental law of gear-tooth action.

### 1.3 Conjugate Action:

Mating gear teeth acting against each other to produce rotary motion are similar to cams. When the tooth profiles, or cams, are designed so as to produce a constant angular-velocity ratio during meshing, these are said to have conjugate action. In theory, at least, it is possible arbitrarily to select any profile for one tooth and then to find a profile for the meshing tooth which will give conjugate action. One of these solutions is the involute profile, which, with few exceptions, is in universal use for gear teeth.

#### There are two forms of tooth profile commonly used:

- Cycloidal teeth
- Involute teeth

An advantage of the Cycloidal teeth over the involute one is that wear of Cycloidal tooth is not as fast as with involute tooth. For this reason, gears transmitting very large amount of power are sometimes cut with Cycloidal teeth.

On the other hand, involute teeth are very easy to manufacture and the actual distance between the centers may deviate slightly from the theoretical distance without affecting the velocity ratio or general performance. Because of this distinct advantage, gears with involute cut teeth are used much more than those with Cycloidal teeth.

**Involute properties:**

There are almost an infinite number of curves that can be developed to satisfy the law of gearing, and many different curve forms have been tried in the past. Modern gearing (except for clock gears) is based on involute teeth. This is due to three major advantages of the involute curve:

- 1 Conjugate action is independent of changes in center distance.
- 2 The form of the basic rack tooth is straight-sided, and therefore is relatively simple and can be accurately made; as a generating tool it imparts high accuracy to the cut gear tooth.
- 3 One cutter can generate all gear teeth numbers of the same pitch.

An involute may be generated as shown in fig. 1.2 a partial flange B is attached to the cylinder A, around which is wrapped a cord xyz which is held tight. Point b on the cord represents the tracing point, and as the cord is wrapped and unwrapped about the cylinder, point b will trace out the involute curve ac.

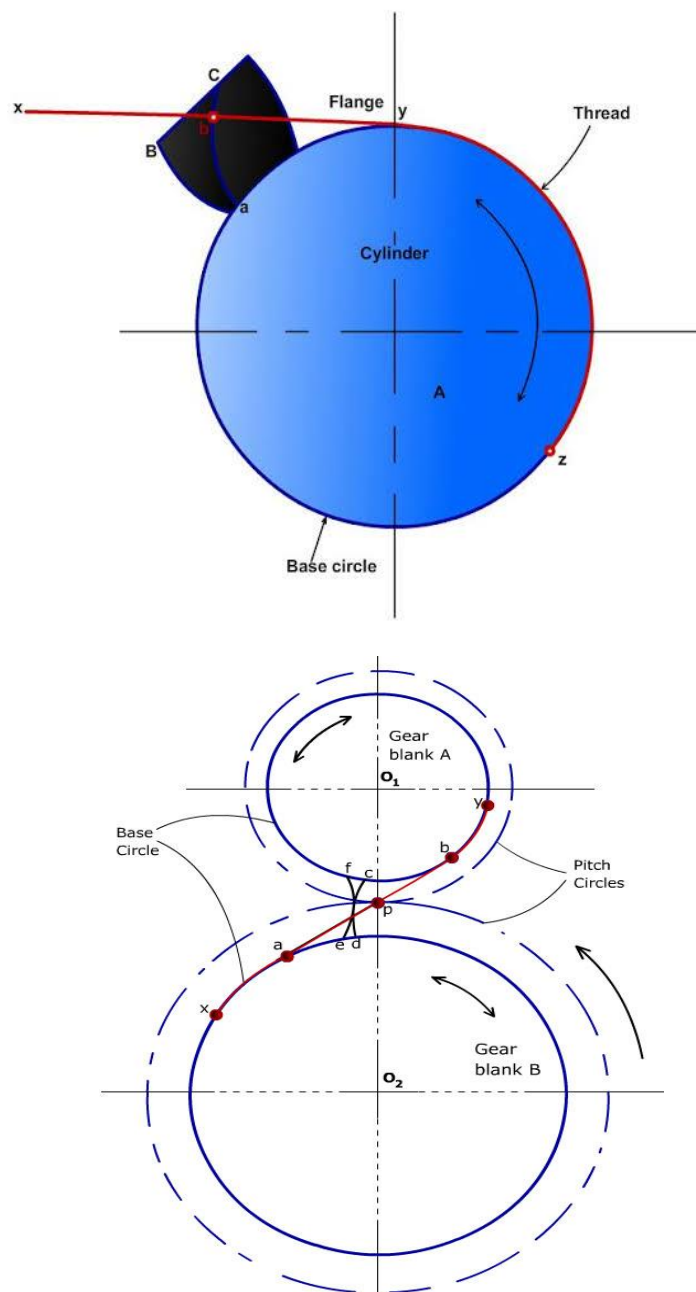


Fig. 1.2 Generation of an involute and involute action

The radius of the curvature of the involute varies continuously, being zero at point 'a' and a maximum at point c. at point b the radius is equal to the distance be, since point "be" is instantaneously rotating about point e. Thus generating line "de" is normal to the involute at all point of intersection and at the same time, is always tangent to the cylinder A. The circle on which the involute is generated is called the base circle.

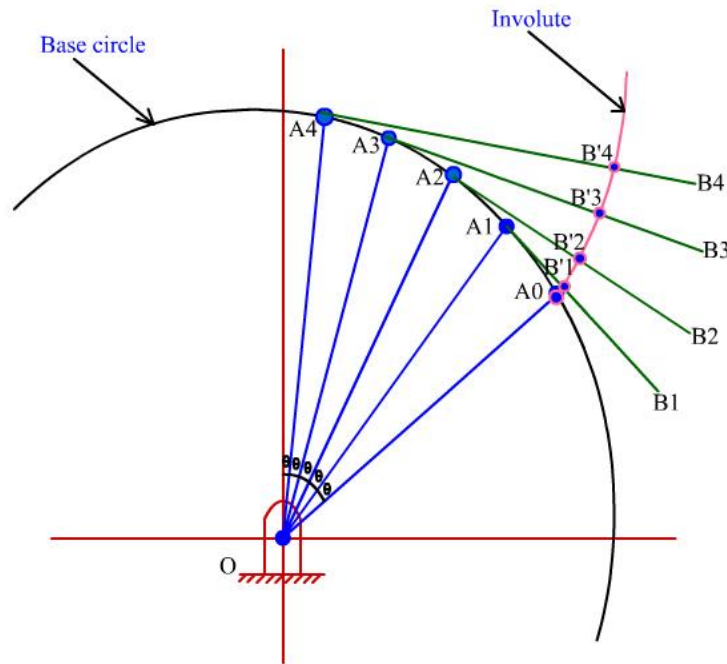


Fig. 1.3 Construction of an Involute curve

## II. LITERATURE REVIEW

**1. OgnyanAlipiev** Proposed new method for the geometric design of symmetric and asymmetric involute meshing in which the contact ratio of the gear drive is equal to its potential. This method is appropriate for the geometric design of spur gears with a small teeth number and is based on the generalized model of involute meshing. The gear drive potential is assigned as an input value, from which, after appropriate calculations, the geometry of rack-cutters, gears and involute meshing can be determined.

**2. V. Senthil Kumar, D.V. Muni and G. Muthuveerappan** generated non-standard asymmetric rack cutters with required pressure angles and module of the required pinion and gear of a drive with asymmetric involute surfaces and trochoidal fillet profiles. The optimization of spur gear set for its center distance, weight and tooth deflections are taken as an objective functions and the decision variable such as module, face width and number of teeth on pinion, and subjected to constraints namely, bending stress, contact stress. Since it is multi-objective function with constraints is very difficult to optimize using conventional optimization techniques, used non-traditional optimization technique called Genetic algorithm.

**3. Alexander Kapelevich** developed the basic geometric theory of the gears with asymmetric teeth. This theory allows to research and design gears independently from generating rack parameters. It also provides wide variety of solutions for a particular couple of gears that are included in the area of existence.

**4. XiangfeiZhao** focus on the geometrical shape of gear tooth fillet profile , usually cut out by the cutter tip , plays a significant role in the evaluation of the gear bending stresses .In order to improve the teeth bending strength ,there search detailed here by introduced novel curve(quadratic rational Bezier curve)to describe the cutter tip .The gear tooth finite element model was founded by APD Linthe ANSYS software. With the maximum bending stress (von Misesstress) as the objective function, sub-problem and first-order optimization method sin ANSYS were used toothimize the cutter tip. The study reveals tha the relationship between the design variable and tooth root bending stress is nonlinear , and the gear cut by the optimized cutter exhibits higher bending strength rather than the gear cut by standard cutting tool.

**5. Prof. G.B.Ingole** deals with study of is to relieve stress from the maximum value to as minimum as possible .So the highest point of contact of teeth is selected pressure application point which causes highest stress Stress relieving feature having a circular shape is used in the path of stress flow which helped to regulate stress flow by redistributing the lines of force. In this study,the best result is obtained by introducing the circular stress relieving feature at centre of tooth.The asymmetric spur gear having circular stress relieving feature at centre of tooth gives better results than the results obtained with simple spurgear having circular stress relieving feature at centre of tooth. The work presented is carried for static condition the method can beextended for dynamic analysis by defining the contact element.

**6. G. Mallesh** The assessment of pressure angle modification on drives side leads to interesting conclusions. While the pressure angle modification affects the gear tooth geometry , the modification study is itself limited by gear parameters like module ,number of teeth ,contact ratio and profile shift .It has been observed that the pressure angle has insignificant influence on the induced stress where as the bending stress is considerably reduced by increasing the pressure angle. Following remarks can also be made regarding pressure angle modification.

**Objective:**

1. To study the effects of pressure angle of gear teeth.
2. To determine the Bending stress induced in asymmetric teeth by experimentally.
3. To prepare a Finite Element model for asymmetric teeth of profile.
4. To solve the finite element model in analysis software &obtain results.
5. To validate the results obtained from experimental readings with FEA model

**III. METHODOLOGY**

1. To find possible solutions.
2. Problem understanding.
3. Selection of preferred solution.
4. Implementation of plan.
5. Data collection.
6. Force and stress analysis.
7. Design of components.
8. Concluding remarks

**IV. TO PLOT THE INVOLUTE CURVE:**

1. To plot the curve in the pro-e .ibl file is required.
- 2 .ibl file contains X, Y and Z co-ordinates of curve.
- 3 to generate .ibl file need to do below procedure:
  - Copy the co-ordinates from the excel file and paste in the notepad file.
  - At the start of the notepad file write below programming codes to convert text file to .ibl file

```
OPEN
ARCLENGTH
BEGIN SECTION! 1
BEGIN CURVE
```

Below this line paste the co-ordinates of the required curve.

For details Refer the .ibl file in the annexure no .-----

- 4 After generation of .ibl file next step is to call .ibl file in the pro-e.
  - a. Open part file in the Pro-e.
  - b. Select the Curve option from the tool bar in that select curve from file
  - c. -Select the generated .ib; file and ok.
  - d. Involute curve is generated.
  - e. Follow the same procedure to plot curve for the coast side, drive side and Torchoidal profile.
  - f. Now the basic curves are generated in the Pro-E. as shown in the Fig.

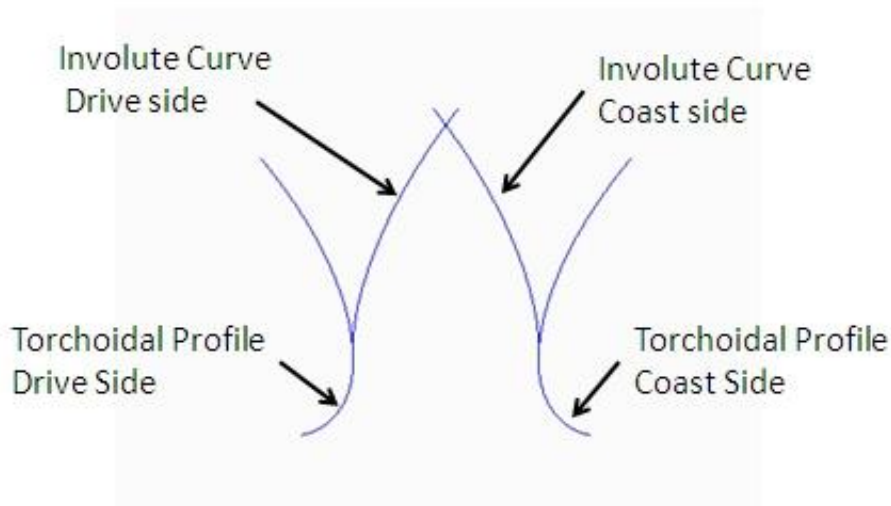


Fig. 4.1 Curves generated in the Pro-E

Now With the help of Outer Diameter, Pitch Circle Diameter, Base Circle Diameter calculated in the excel file. Generate the tooth profile as shown in fig.

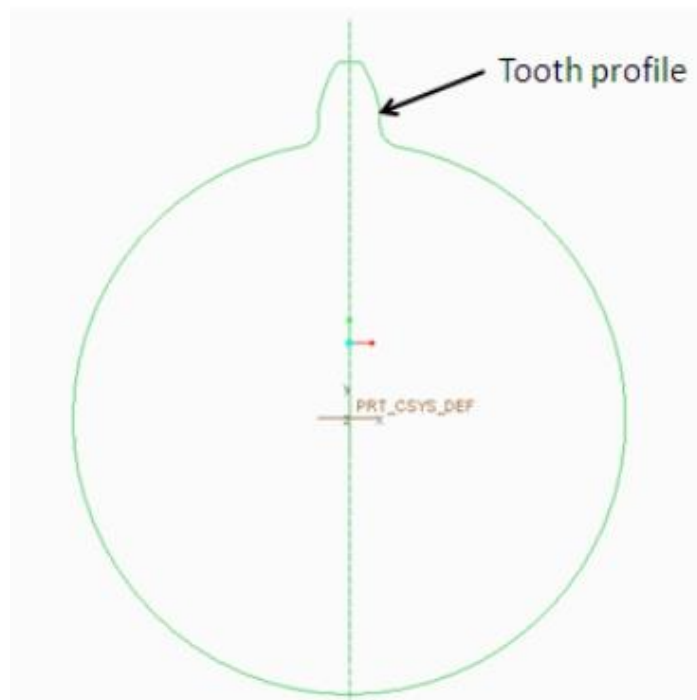


Fig. 4.2 Sketch of gear teeth

With help of this sketch generate the 3D Model.

- Now the gear is ready.
- In this study, For the analysis only one teeth is considered, so model as shown in the fig5.3 is generated.

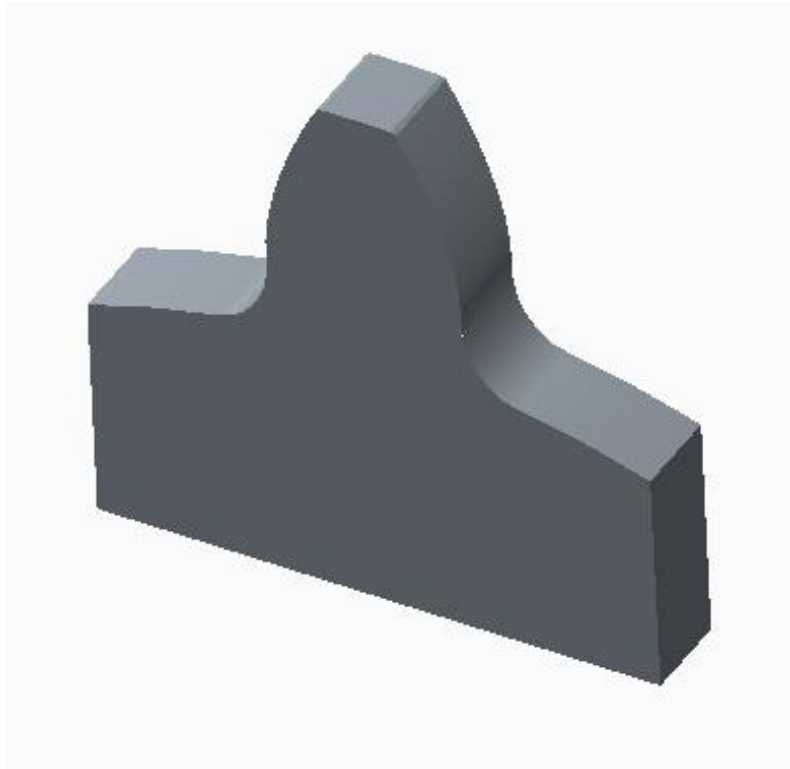


Fig. 4.3 3D model of Gear Teeth

Save the CAD model in the .step file so that it can be imported for the analysis for the further calculations.

## V. FEA ANALYSIS OF THE BENDING STRESS IN THE GEAR TOOTH

1. Pre-processing:

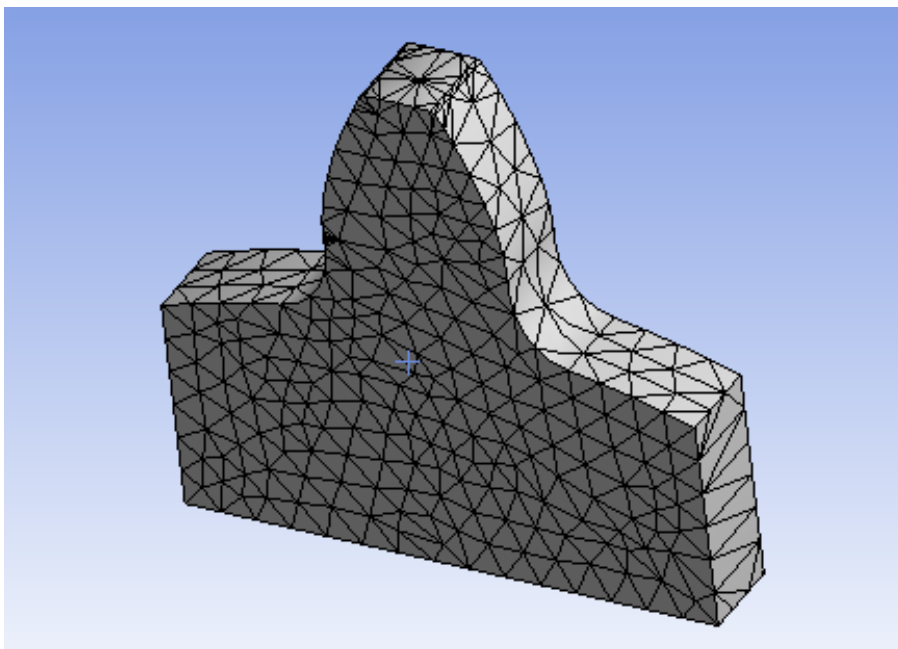


Fig. 5.1 Tooth Meshing

2. Post-Processing:

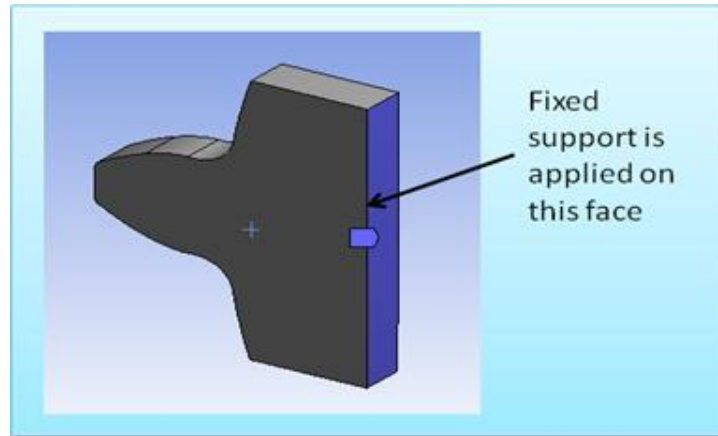


Fig. 5.2 Fixed Support

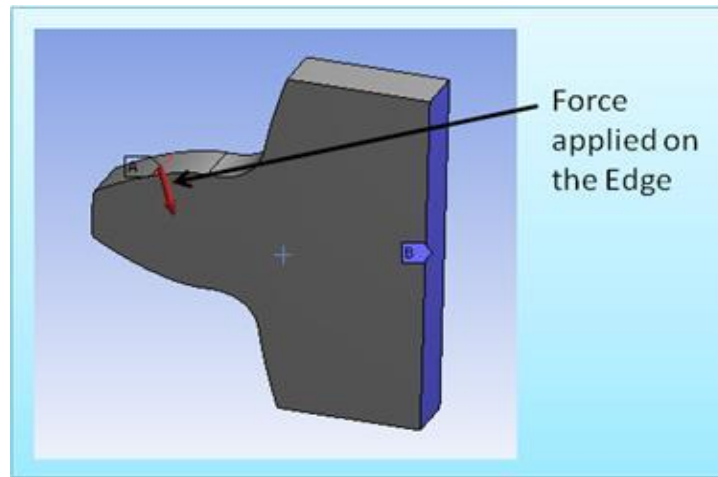


Fig. 5.3 force applied

- After getting solution it is concluded that the bending load carrying capacity of the asymmetric gear is more than the symmetric gear.

We get 8 max .stress result by 8 specimen from that one result given below

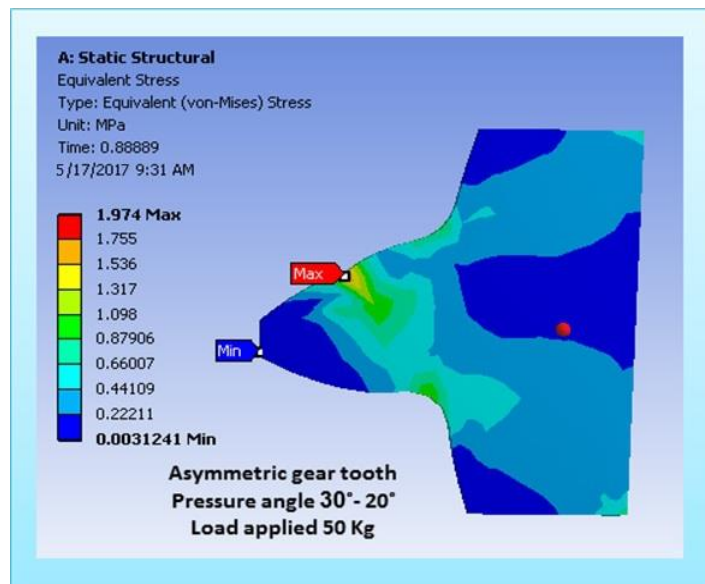


Fig. 5.4 test specimen



## VI. EXPERIMENTAL STRESS TEST

In this study, for the testing of bending stress in the gear tooth “Diffused light polariscope” is used.

Diffused Light Polariscope:-



Fig. 6.1 Diffused light Polariscope

Photoelasticity is a nondestructive, whole-field, graphic stress-analysis technique based on an optomechanical property called birefringence, possessed by many transparent polymers. Combined with other optical elements and illuminated with an ordinary light source, a loaded photoelastic specimen (or photoelastic coating applied to an ordinary specimen) exhibits fringe patterns that are related to the difference between the principal stresses in a plane normal to the light propagation direction.

The method is used primarily for analyzing two dimensional plane problems, which is the emphasis in these notes. A method called stress freezing allows the method to be extended to three dimensional problems. Photoelastic coatings are used to analyze surface stresses in bodies of complex geometry.

### Advantages and disadvantages:

#### *Advantages:*

Photoelasticity, as used for two-dimensional plane problems,

- provides reliable full-field values of the difference between the principal normal stresses in the plane of the model
- provides uniquely the value of the non vanishing principal normal stress along the perimeter(s) of the model, where stresses are generally the largest
- furnishes full-field values of the principal stress directions (sometimes called stress trajectories)
- is adaptable to both static and dynamic investigations
- requires only a modest investment in equipment and materials for ordinary work
- is fairly simple to use

#### *Disadvantages:*

On the other hand, photoelasticity requires that a model of the actual part be made (unless photoelastic coatings are used)

- requires rather tedious calculations in order to separate the values of principal stresses at a general interior point
- can require expensive equipment for precise analysis of large components
- is very tedious and time-consuming for three-dimensional work

**Procedure:**

The procedure for preparing two-dimensional models from pre-machined templates will be described. Alternatively, specimens may be machined “from scratch,” in which case a computer controlled laser cutting machine is recommended.

**1. Selecting the material.** Many polymers exhibit sufficient birefringence to be used as photoelastic specimen material. However, such common polymers as polymethylmethacrylate (PMMA) and polycarbonate may be either too brittle or too intolerant of localized straining. Homalite®-100 has long been a popular general purpose material, available in various thicknesses in large sheets of optical quality. Perspex acrylic is a more recently introduced material that has excellent qualities, both for machining and for fringe sensitivity. Another good material is epoxy, which may be cast between plates of glass, but this procedure is seldom followed for two-dimensional work.

**2. Making a template.** If more than 2 or 3 pieces of the same shape are to be made, it is advisable to machine a template out of metal first. This template may then be used to fabricate multiple photoelastic specimens having the same shape as that of the template. The template should be undercut by about 0.050 in. through about half the template thickness from one side to avoid contact with the router bit (explained below).

**3. Machining the specimen.** If the specimen is machined “from scratch,” care must be taken to take very light cuts with a sharp laser cutter in order to avoid heating the specimen unduly along its finished edges. A coolant, such as ethyl alcohol, kerosene, or water, should be used to minimize heating.

**4. Viewing the loaded specimen.** After the specimen is removed from the template and cleaned, it is ready for loading. A polariscope (to be described later) is needed for viewing the fringes induced by the stresses. The elements of the polariscope must be arranged so as to allow light to propagate normal to the plane of the specimen.

**5. Recording the fringe patterns.** An ordinary still camera or a video camera may be used to record the fringe patterns.

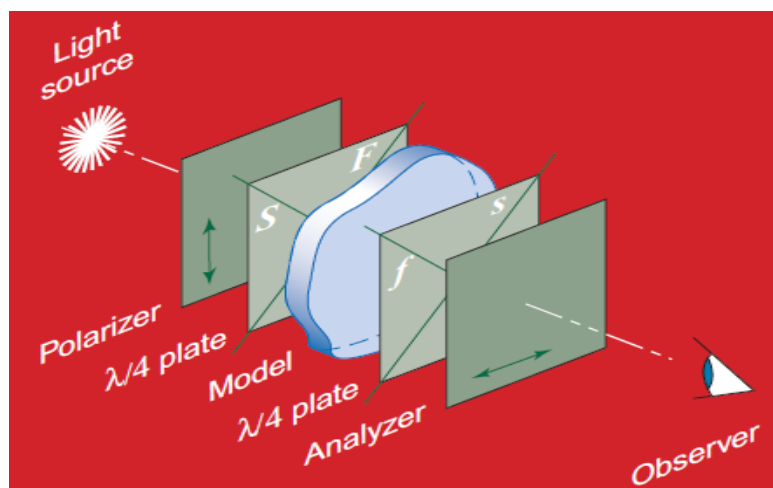
**6. Interpreting the fringe patterns.** Two types of pattern can be obtained: iso chromatics and iso clinics. These patterns are related to the principal-stress differences and to the principal stress directions, respectively.

**Polarization:**

A given light wave has an amplitude vector that is always perpendicular to its propagation direction. However, for ordinary light, the orientation of the amplitude vector in the plane perpendicular to the propagation direction is totally random.

**Plane-polarized light:**

If the amplitude vectors of all light rays emanating from a source are restricted to a single plane, as in Fig. 5.2, the light is said to be plane polarized. An observer viewing the light wave head-on would see the wave with its amplitude vector restricted to a single plane, which is called the plane of polarization.



**Fig. 6.2 Light Propagation**

In this Study, for the experimental stress analysis 8 component of Perspex acrylic is manufactured. . All the gear tooth are machined by using laser cutting machine.

**Calculation of stress by photoelastic method:**

A vertical force P acts on a horizontal straight boundary of an infinitely large plate Fig. 5.4 The stress function, (Timoshenko and Goodier, 1970), is given as:

$$\phi = -\left(\frac{P}{\pi}\right) \cdot r \cdot \theta \cdot \sin\theta \quad \dots\dots\dots 1$$

The stresses  $\sigma_r, \sigma_\theta, \sigma_{r\theta}$  are given by the relations:

$$\sigma_r = \left(\frac{1}{r}\right) \left(\frac{\partial\phi}{\partial r}\right) + \left(\frac{1}{r^2}\right) \left(\frac{\partial^2\phi}{\partial\theta^2}\right) = -\left(\frac{2P\cos\theta}{\pi \cdot r}\right) \quad \dots\dots\dots 2$$

$$\sigma_\theta = \frac{\partial^2\phi}{\partial r^2} = 0 \quad \dots\dots\dots 3$$

$$\sigma_{r\theta} = -\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial\phi}{\partial\theta}\right) = 0 \quad \dots\dots\dots 4$$

According to the stress-optical law the difference of the principal stresses is:

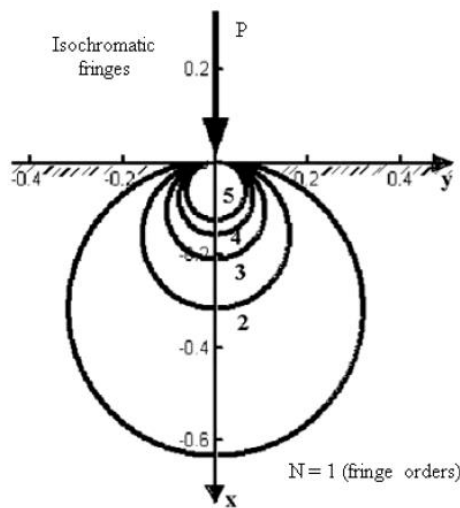
$$\sigma = \sigma_1 - \sigma_2 = \frac{N \cdot f \cdot \sigma}{d} \quad \dots\dots\dots 5$$

Where:

N = The isochromatic fringe order

d = The thickness of the specimens

fσ = The material fringe value or stress-optical constant



**Fig. 6.3 Isochromatic fringe patterns**

Testing procedure:-

- Fixture is designed as shown in the fig to hold the job for the testing.
- Set the job at required position as shown in fig.
- As shown in the snap arrange the load point as per requirement.
- Balance the weight bar, so that applied load can be easily measured.
- Apply the load gradually in the load tray as shown in the fig.
- Now at required load you will be able to see the fringe patterns developed in the test specimen.
- In this study 8 specimen are tested at different load condition, for details refer the below figs.
- In that 4 samples are tested at 50kg load and 4 samples are tested at 35kg load as shown in the fig.
- From the below fig comparison of bending stress development between the symmetric and the asymmetric gear tooth is done.

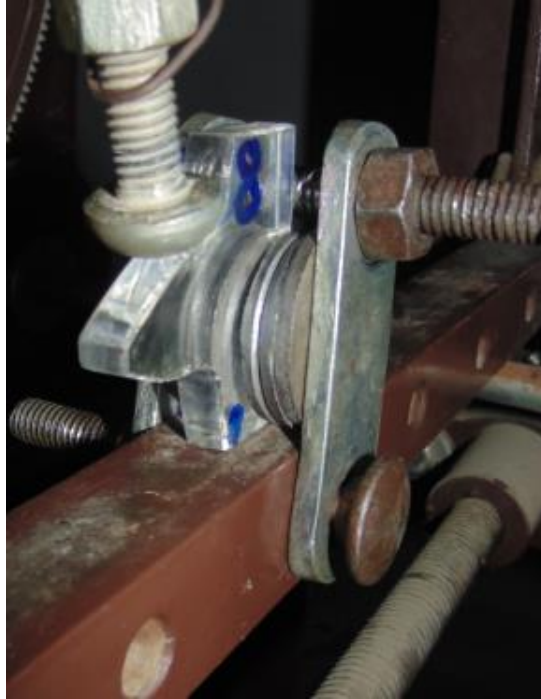


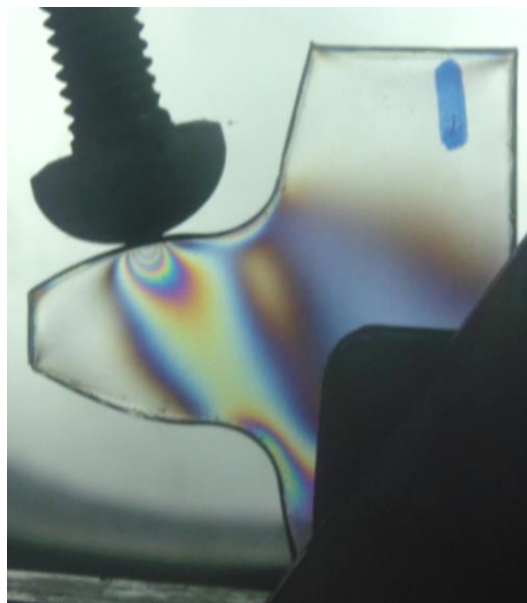
Fig. 6.4 Fixture to hold the Job

- From the below fig it is observed that bending stress gets reduced in the asymmetric gear than the symmetric gear at same load condition.

## VII. RESULTS AND DISCUSSION

From the below experimental results it is prove that, it is possible to increase the bending load carrying capacity of spur gear by modifying symmetric geometry to Asymmetric geometry as explained in this study.

For the Details refer the below results

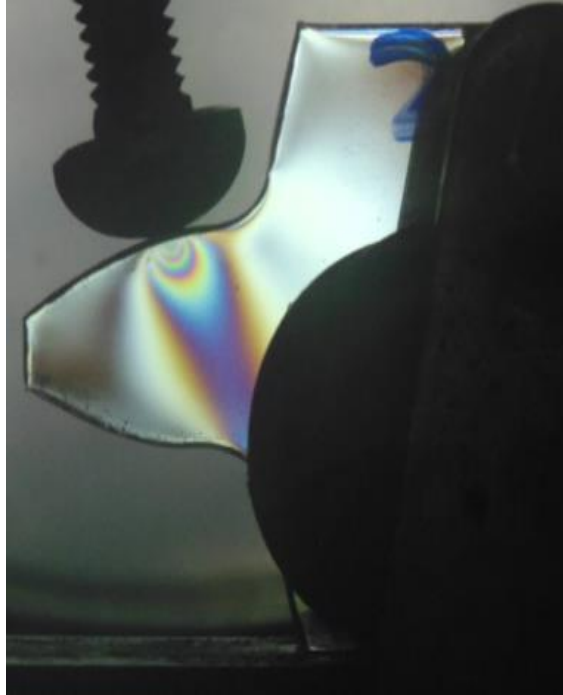


Symmetric gear

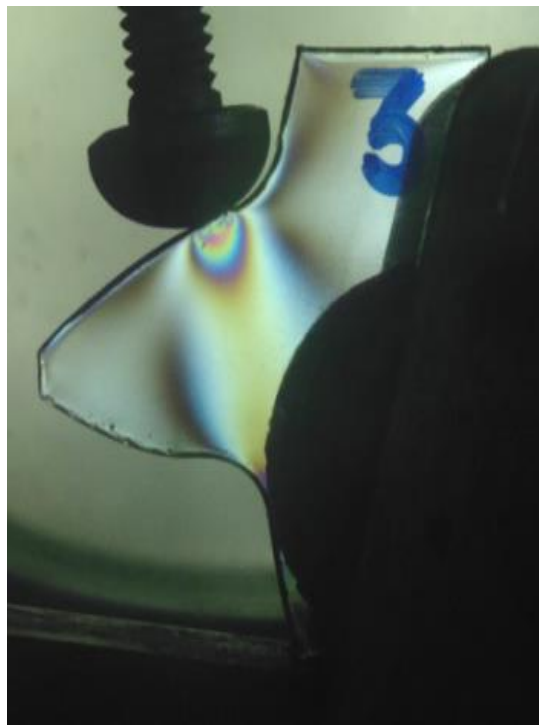
Pressure angle  $20^{\circ}$ - $20^{\circ}$

Load applied = 50 kg

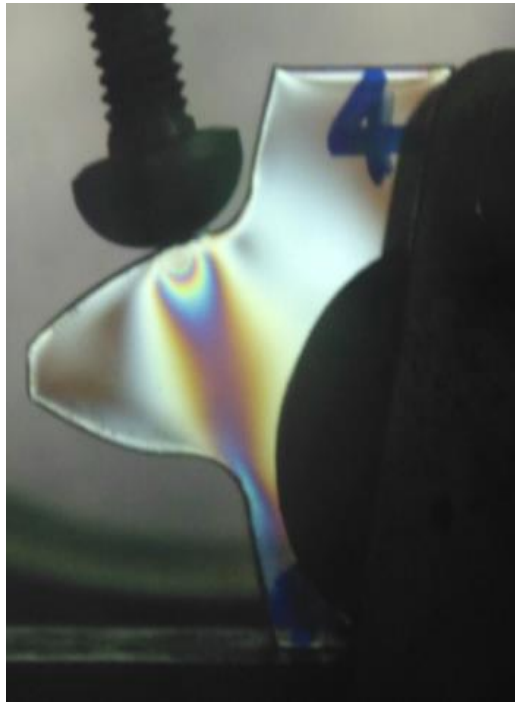
Test Specimen 1



Asymmetric gear  
Pressure angle  $25^{\circ}$ - $20^{\circ}$   
Load applied = 50 kg  
Test Specimen 2



Asymmetric gear  
Pressure angle  $35^{\circ}$ - $20^{\circ}$   
Load applied = 50 kg  
Test Specimen 3



Asymmetric gear  
Pressure angle  $30^{\circ}$ - $20^{\circ}$   
Load applied = 50 kg  
Test Specimen 4



Symmetric gear  
Pressure angle  $20^{\circ}$ - $20^{\circ}$   
Load applied = 35 kg  
Test Specimen 5

Stress calculation by experimental method is as below:-

$$\sigma = \frac{N.F\sigma}{d} = \frac{5 * 4.4768}{8}$$

$$\sigma = 2.798 \text{ Mpa}$$

**Table 6.5: Experimental estimations of maximum stresses of the specimens**

Test specimen	Isocromatic order	Width of sample	Maximum stress( $\sigma$ ) MPa
1	5	8	2.798
2	4	8	2.2384
3	2	8	1.1192
4	2	8	1.1192
5	3	8	1.6788
6	3	8	1.6788
7	2	8	1.1192
8	2	8	1.1192

## VIII. RESULTS

**Table 7.1: Deviation between Experimental and FEA**

Test specimen	Estimation of Maximum stress ( $\sigma$ ) MPa by Photoelastic analysis	Estimation of Maximum stress ( $\sigma$ ) MPa by FEA	% Deviation between Experimental and FEA
1	2.798	2.876	2.78
2	2.2384	2.152	-3.85
3	1.6788	1.974	1.58
4	1.6788	1.718	2.33
5	1.6788	1.6649	-0.82
6	1.1192	1.273	1.74
7	1.1192	1.152	2.93
8	1.1192	1.162	3.82

If we compare the results in Table , we can see that there is avg. deviation of the maximum stress estimation from -3.8% to +3.8% between the Estimation of Maximum stress ( $\sigma$ ) MPa by Photoelastic analysis and the Estimation of Maximum stress ( $\sigma$ ) MPa by FEA. This deviation increases with the number of teeth of the bigger gear, keeping constant the number of pinion teeth.

In this study, the minimum deviation between the results of the applied methods was investigated. Results of photoelasticity experiments, which is the most widely applied experimental method for gear stress analysis, were compared to the results of the finite elements method using ANSYS software. Comparison of the results of the two applied methods proved that the deviations are acceptable. These deviations are as on able considering the potential errors that can be involved during the application of the two methods.

## IX. PROBLEM AND ADVANTAGES

### A. Problems:

Here, we present a novel methodology for “Design of involute spur gear with asymmetric teeth to improve bending Load carrying capacity of the Spur Gear.” To evaluate this we have to apply the experimental method by using FEA analysis .Here also we have to use analysis software for evaluating the bending stress by taking various pressure angle. After getting the results from experimental method and analysis software, results can be validated by both ways and bending stress can be minimized by different considerable factors such as pressure angle.

**B. Advantages:**

- Higher load capacity
- Reduced size and weight
- Extended lifetime
- Reduced noise and vibration
- Higher efficiency
- Higher reliability
- Reduced cost

**X. CONCLUSION**

The basic geometric theory of the gears with asymmetric teeth has been developed. This theory allows to research and design gears independently from generating rack parameters. It also provides wide variety of solutions for a particular couple of gears that are included in the area of existence. The following conclusion can be drawn from this research:

The asymmetric tooth geometry allows for an increase in load capacity while reducing weight and dimn. For some types of gears. It becomes possible by increasing of the pressure angle and constant ratio for drive sides

It is possible to increase the bending load carrying capacity of spur gear by modifying symmetric geometry to Asymmetric geometry by FEA Analysis.

**ACKNOWLEDGMENT**

First and foremost, I would like to express my deep sense of gratitude and indebtedness to my Guide and PG coordinator Dr. R.R. Navthar Sir, Department of Mechanical Engineering for his invaluable encouragement, suggestions and support from an early stage of this Project and providing me extraordinary experiences throughout the work. I am highly grateful to the Principal, Dr. J..Jayakumar and Head of Department of Mechanical Engineering, Dr. K.B.Kale, PDVVP COE, Ahmednagar for their kind support and permission to use the facilities available in the Institute..

**REFERENCES**

- [1] Ognyan Alipiev “Geometric design of involute spur gear drives with symmetric and asymmetric teeth using the Realized Potential Method”, *Mechanism and Machine Theory* 46 2011 10–32 September 2010 98
- [2] G.Malles , Dr.V.B.Math, Shiva Prasad“Parametric analysis of Asymmetric Spur Gear Tooth” *14th National Conference on Machines and Mechanisms (NaCoMM09)*, NIT, Durgapur, India, December 17-18, 2009
- [3] Alexander Kapelevich “Geometry and design of involute spur gears with asymmetric teeth”, *Gear Technology*, Received 27 April 1998; received in revised form 15 October 1998; accepted 23 October 1998
- [4] E. B. Vulgakov “Gears with Improved Characteristics”,1974, Mashinostroenie, Moscow (in Russian).
- [5] A. L. Kapelevich, R. E. Kleiss,Gear “Direct Gear Design for Spur and Helical Involutess Gears”, *Gear technology*, September/October 2002
- [6] Dr.Safaa H. Abdulrahman, Dr. Adnan D. Mohammed “An Approach For Forming Spur Gear Tooth Profile”. *Eng.&Tech*.Vol.26,No.8,2008
- [7] *Townsend D.P. Dudley’s Gear Handbook*, McGraw-Hill, 1991.
- [8] Handbook of gear Design Gitin M Maitra.